

Anexo D

Tabla de Integrales

(PUEDE SUMARSE UNA CONSTANTE ARBITRARIA A CADA INTEGRAL)

$$1. \int x^n dx = \frac{1}{n+1} x^{n+1} \quad (n \neq -1)$$

$$2. \int \frac{1}{x} dx = \log |x|$$

$$3. \int e^x dx = e^x$$

$$4. \int a^x dx = \frac{a^x}{\log a}$$

$$5. \int \operatorname{sen} x dx = -\cos x$$

$$6. \int \cos x dx = \operatorname{sen} x$$

$$7. \int \tan x dx = -\log |\cos x|$$

$$8. \int \cot x dx = \log |\operatorname{sen} x|$$

$$9. \int \sec x dx = \log |\sec x + \tan x| = \log \left| \tan \left(\frac{1}{2} x + \frac{1}{4} \pi \right) \right|$$

10. $\int \csc x \, dx = \log |\csc x - \cot x| = \log \left| \tan \frac{1}{2} x \right|$
11. $\int \arcsen \frac{x}{a} \, dx = x \arcsen \frac{x}{a} + \sqrt{a^2 - x^2} \quad (a > 0)$
12. $\int \arccos \frac{x}{a} \, dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} \quad (a > 0)$
13. $\int \arctan \frac{x}{a} \, dx = x \arctan \frac{x}{a} - \frac{a}{2} \log (a^2 + x^2) \quad (a > 0)$
14. $\int \sen^2 mx \, dx = \frac{1}{2m} (mx - \sen mx \cos mx)$
15. $\int \cos^2 mx \, dx = \frac{1}{2m} (mx + \sen mx \cos mx)$
16. $\int \sec^2 x \, dx = \tan x$
17. $\int \csc^2 x \, dx = -\cot x$
18. $\int \sen^n x \, dx = -\frac{\sen^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sen^{n-2} x \, dx$
19. $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sen x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
20. $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx \quad (n \neq 1)$
21. $\int \cot^n x \, dx = \frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx \quad (n \neq 1)$
22. $\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx \quad (n \neq 1)$
23. $\int \csc^n x \, dx = \frac{\cot x \csc^{n-1} x}{n-2} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx \quad (n \neq 1)$
24. $\int \sinh x \, dx = \cosh x$
25. $\int \cosh x \, dx = \sinh x$

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26. $\int \tanh x \, dx = \log |\cosh x|$
27. $\int \coth x \, dx = \log |\operatorname{sen} hx|$
28. $\int \operatorname{sech} x \, dx = \arctan (\operatorname{senh} x)$
29. $\int \operatorname{csch} x \, dx = \log \left| \tanh \frac{x}{2} \right| = -\frac{1}{2} \log \frac{\cosh x + 1}{\cosh x - 1}$
30. $\int \operatorname{senh}^2 x \, dx = \frac{1}{4} \operatorname{senh} 2x - \frac{1}{2} x$
31. $\int \operatorname{cosh}^2 x \, dx = \frac{1}{4} \operatorname{senh} 2x + \frac{1}{2} x$
32. $\int \operatorname{sech}^2 x \, dx = \tanh x$
33. $\int \operatorname{senh}^{-1} \frac{x}{a} \, dx = x \operatorname{senh}^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} \quad (a > 0)$
34. $\int \operatorname{cosh}^{-1} \frac{x}{a} \, dx = \begin{cases} x \operatorname{cosh}^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} & \left[\operatorname{cosh}^{-1} \left(\frac{x}{a} \right) > 0, a > 0 \right] \\ x \operatorname{cosh}^{-1} \frac{x}{a} + \sqrt{x^2 - a^2} & \left[\operatorname{cosh}^{-1} \left(\frac{x}{a} \right) < 0, a > 0 \right] \end{cases}$
35. $\int \operatorname{tanh}^{-1} \frac{x}{a} \, dx = x \operatorname{tanh}^{-1} \frac{x}{a} + \frac{a}{2} \log |a^2 - x^2|$
36. $\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \log \left(x + \sqrt{a^2 + x^2} \right) = \operatorname{sen} h^{-1} \frac{x}{a} \quad (a > 0)$
37. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{2} \arctan \frac{x}{a} \quad (a > 0)$
38. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arcsen} \frac{x}{a} \quad (a > 0)$
39. $\int (a^2 - x^2)^{\frac{3}{2}} \, dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \operatorname{arcsen} \frac{x}{a} \quad (a > 0)$
40. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \operatorname{arcsen} \frac{x}{a} \quad (a > 0)$
41. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right|$

42. $\int \frac{1}{(a^2 - x^2)^{\frac{3}{2}}} dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
43. $\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log \left| x + \sqrt{x^2 \pm a^2} \right|$
44. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| = \cosh^{-1} \frac{x}{a} \quad (a > 0)$
45. $\int \frac{1}{x(a + bx)} dx = \frac{1}{a} \log \left| \frac{x}{a + bx} \right|$
46. $\int x \sqrt{a + bx} dx = \frac{2(3bx - 2a)(a + bx)^{\frac{3}{2}}}{15b^2}$
47. $\int \frac{\sqrt{a + bx}}{x} dx = 2\sqrt{a + bx} + a \int \frac{1}{x\sqrt{a + bx}} dx$
48. $\int \frac{x}{\sqrt{a + bx}} dx = \frac{2(bx - 2a)\sqrt{a + bx}}{3b^2}$
49. $\int \frac{1}{x\sqrt{a + bx}} dx = \begin{cases} \frac{1}{\sqrt{a}} \log \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| & (a > 0) \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a+bx}{-a}} & (a < 0) \end{cases}$
50. $\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \log \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
51. $\int x \sqrt{a^2 - x^2} dx = -\frac{1}{3} (a^2 - x^2)^{\frac{3}{2}}$
52. $\int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsen \frac{x}{a} \quad (a > 0)$
53. $\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \log \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
54. $\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$
55. $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsen \frac{x}{a} \quad (a > 0)$
56. $\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \log \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$

$$57. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \arccos \frac{a}{|x|} = \sqrt{x^2 - a^2} - \operatorname{arcsec} \left(\frac{x}{a} \right) \quad (a > 0)$$

$$58. \int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{\frac{3}{2}}$$

$$59. \int \frac{1}{x\sqrt{x^2 + a^2}} dx = \frac{1}{a} \log \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right|$$

$$60. \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \arccos \frac{a}{|x|} \quad (a > 0)$$

$$61. \int \frac{1}{x^2\sqrt{x^2 \pm a^2}} dx = \pm \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

$$62. \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$$

$$63. \int \frac{1}{ax^2 + bx + c} dx = \begin{cases} \frac{1}{\sqrt{b^2 - 4ac}} \log \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| & (b^2 > 4ac) \\ \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & (b^2 < 4ac) \end{cases}$$

$$64. \int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \log |ax^2 + bx + c| - \frac{b}{2a} \int \frac{1}{ax^2 + bx + c} dx$$

$$65. \int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \begin{cases} \frac{1}{\sqrt{a}} \log |2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}| & (a > 0) \\ \frac{1}{\sqrt{-a}} \operatorname{arcsen} \frac{-2ax - b}{\sqrt{b^2 - 4ac}} & (a < 0) \end{cases}$$

$$66. \int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$$

$$67. \int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$$

$$68. \int \frac{1}{x\sqrt{ax^2 + bx + c}} dx = \begin{cases} \frac{-1}{\sqrt{c}} \log \left| \frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x} \right| & (c > 0) \\ \frac{1}{\sqrt{-c}} \operatorname{arcsen} \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} & (c < 0) \end{cases}$$

$$69. \int x^3\sqrt{x^2 + a^2} dx = \left(\frac{1}{5}x^2 - \frac{2}{15}a^2 \right) \sqrt{(a^2 + x^2)^3}$$

$$70. \int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{\sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}$$

$$71. \int \operatorname{sen} ax \operatorname{sen} bx dx = \frac{\operatorname{sen}(a - b)x}{2(a - b)} - \frac{\operatorname{sen}(a + b)x}{2(a + b)} \quad (a^2 \neq b^2)$$

$$72. \int \operatorname{sen} ax \cos bx \, dx = \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$73. \int \cos ax \cos bx \, dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$74. \int \sec x \tan x \, dx = \sec x$$

$$75. \int \csc x \cot x \, dx = -\csc x$$

$$76. \int \cos^m x \operatorname{sen}^n x \, dx = \frac{\cos^{m-1} x \operatorname{sen}^{n-1} x + x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \operatorname{sen}^n x \, dx = \\ = -\frac{\operatorname{sen}^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \operatorname{sen}^{n-2} x \, dx$$

$$77. \int x^n \operatorname{sen} ax \, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$78. \int x^n \cos ax \, dx = \frac{1}{a} x^n \operatorname{sen} ax - \frac{n}{a} \int x^{n-1} \operatorname{sen} ax \, dx$$

$$79. \int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

$$80. \int x^n \log(ax) \, dx = x^{n+1} \left[\frac{\log ax}{n+1} - \frac{1}{(n+1)^2} \right]$$

$$81. \int x^n (\log ax)^m \, dx = \frac{x^{n+1}}{n+1} (\log ax)^m - \frac{m}{n+1} \int x^n (\log ax)^{m-1} \, dx$$

$$82. \int e^{ax} \operatorname{sen} bx \, dx = \frac{e^{ax} (a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2}$$

$$83. \int e^{ax} \cos bx \, dx = \frac{e^{ax} (b \operatorname{sen} bx + a \cos bx)}{a^2 + b^2}$$

$$84. \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$$

$$85. \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x$$